### Degree competences to which the subject contributes

**Specific:**
- G1. Ability to solve arithmetic problems related to engineering. Aptitude to apply knowledge concerning: linear algebra, geometry, differential geometry, differential and integral calculus, numerical methods, statistics technology.

**Transversal:**
- 2. SELF-DIRECTED LEARNING. Detecting gaps in one's knowledge and overcoming them through critical self-appraisal. Choosing the best path for broadening one's knowledge.
- 4. EFFECTIVE USE OF INFORMATION RESOURCES. Managing the acquisition, structure, analysis and display of information from the own field of specialization. Taking a critical stance with regard to the results obtained.

### Teaching methodology

There are large group classes, that deal with theoretical explanations, descriptions of selected examples and problem solving. In the computer lab sessions, students perform simulations with computer software in order to solve case-studies.

### Learning objectives of the subject

* To understand the concepts and techniques of classical geometry that are essential for CAGD:
  - To use affine coordinates and transformations to move and transform the shape of plane and spacial geometric figures
  - To handle with conics and quadric surfaces, as example of basic curves and surfaces
  - To identify some affine, Euclidean and projective characteristics (barycentric coordinates, distance, cross ratio)
  - To understand the following concepts of differential geometry: curvature, torsion and osculating circle of a curve; tangent plane, normal vector and Dupin indicatrix of a surface

* To use the techniques of Bézier and B-splines in designing curves and surfaces:
  - To deal with Bernstein polynomials for Bézier curves and surfaces
  - To learn the de Casteljau Algorithm
  - To understand the problem of geometric continuity for spline curves and surfaces
  - To complete the knowledge of Calculus that is necessary to achieve the previous goals
# Mathematics for Design

## Study load

<table>
<thead>
<tr>
<th>Total learning time: 150h</th>
<th>Hours large group: 45h</th>
<th>30.00%</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Hours medium group: 0h</td>
<td>0.00%</td>
</tr>
<tr>
<td></td>
<td>Hours small group: 15h</td>
<td>10.00%</td>
</tr>
<tr>
<td></td>
<td>Guided activities: 0h</td>
<td>0.00%</td>
</tr>
<tr>
<td></td>
<td>Self study: 90h</td>
<td>60.00%</td>
</tr>
</tbody>
</table>
# Content

## 1. Affine and Euclidean Geometry

**Learning time:** 42h

- Theory classes: 16h
- Laboratory classes: 4h
- Self study: 22h

**Description:**

- 1.1 Points and vectors
- 1.2 Review of linear spaces: linear combinations and basis
- 1.3 Affine space and affine references
- 1.4 Affine combinations, barycentric coordinates and ratio of 3 collinear points
- 1.5 Affine maps
- 1.6 Metric, norm and distance; Euclidean affine space
- 1.7 Orthonormal references
- 1.8 Euclidean motions

## 2. Bézier curves and B-splines curves

**Learning time:** 30h

- Theory classes: 8h
- Laboratory classes: 4h
- Self study: 18h

**Description:**

- 2.1 Linear interpolation. Examples of curves
- 2.2 Bernstein Polynomials
- 2.3 Bézier curves. Properties
- 2.4 Algorithm of de Casteljau
- 2.5 Geometric continuity
- 2.6 B-spline curves

## 3. Rational curves

**Learning time:** 28h

- Theory classes: 8h
- Laboratory classes: 2h
- Self study: 18h

**Description:**

- 3.1 Projective geometry: Conics, Projections, Crossratio
- 3.2 Rational Bézier curves and NURBS
### 4. Differential Geometry of curves

**Description:**
- 4.1 Regular parametrizations
- 4.2 Curvature and torsion
- 4.3 Osculating circle and evolutes
- 4.4 Frenet frame
- 4.5 Geometric continuity

**Learning time:** 24h
- Theory classes: 6h
- Laboratory classes: 2h
- Self study: 16h

### 5. Surfaces

**Description:**
- 5.1 Functions of two variables: continuity, differentiability, Jacobian matrix and chain rule
- 5.2 Differential geometry of surfaces: tangent plane, Gaussian curvature, Dupin's indicatrix
- 5.3 Surfaces of revolution
- 5.4 Rectangular Bézier surfaces

**Learning time:** 24h
- Theory classes: 8h
- Self study: 16h
Planning of activities

**1: EXAM OF ITEMS 1 AND 2 (FIRST MIDTERM EXAM)**

**Description:**
Exam: Problems and theoretical questions of topics 1 and 2

**2: EXAM OF ITEMS 3, 4 AND 5 (SECOND MIDTERM EXAM)**

**Description:**
Exam: Problems and theoretical questions of topics 3, 4 and 5

**3: COMPUTER LAB**

**Description:**
Students should apply basic techniques to handling and depict geometric objects (curves and surfaces), both in general and by Bézier techniques in particular. They will use the software MAPLE ®

**4: FINAL EXAM**

**Description:**
Exam: Problems and theoretical questions of topics 1, 2, 3, 4 and 5

Qualification system

\[
\max((0.35 \times NA1+0.35 \times NA2+0.3 \times NA3),(0.7 \times NA4 + 0.3 \times NA3))
\]

NA1: First midterm exam (activity 1)
NA2: Second midterm exam (activity 2)
NA3: Computer Lab reports for case studies (activity 3)
NA4: Final exam (activity 4)

The completion of activity 3 is a necessary condition to be evaluated
340070 - MADI-D2O43 - Mathematics for Design

Bibliography

Basic:


Complementary:


