Degree competences to which the subject contributes

Specific:
1. CE1. Ability to solve arithmetic problems related to engineering. Aptitude to apply knowledge concerning: linear algebra, geometry, differential geometry, differential and integral calculus, differential and partial equations, numerical methods, numerical algorithms, statistics and optimization.

Transversal:
2. SELF-DIRECTED LEARNING - Level 1. Completing set tasks within established deadlines. Working with recommended information sources according to the guidelines set by lecturers.
3. EFFECTIVE USE OF INFORMATION RESOURCES - Level 1. Identifying information needs. Using collections, premises and services that are available for designing and executing simple searches that are suited to the topic.

Teaching methodology
In the lectures the instructor presents some motivating ideas, the fundamental concepts and some relevant developments, intermingled with key examples and the resolution of representative problems. In some sessions, one or more problems can be assigned to be individually solved in the classroom and to be graded.

In the lab classes the students learn how to solve some problems numerically, using MATLAB. These will be graded by means of written homework.

Learning objectives of the subject
340025 - EQDI-F2O43 - Differential Equations

Calculate the Fourier series of a given periodic function, both in real and complex form, and represent and analyze its spectrum.

Understand the role of differential equations in engineering and science, and see some examples.

Compute the solution of the simplest classes of differential equations of first and second order in the time or Laplace domains, and use the appropriate numerical methods when necessary.

Compute and explain some characteristics, quantities and functions related to linear differential equations with constant coefficients, such as stability, characteristic time or transfer functions.

<table>
<thead>
<tr>
<th>Study load</th>
<th>Hours large group:</th>
<th>Hours medium group:</th>
<th>Hours small group:</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Total learning time:</strong> 150h</td>
<td>52h 30m</td>
<td>0h</td>
<td>7h 30m</td>
</tr>
<tr>
<td>Hours small group:</td>
<td></td>
<td>0h</td>
<td>5.00%</td>
</tr>
<tr>
<td>Guided activities:</td>
<td></td>
<td>0h</td>
<td>0.00%</td>
</tr>
<tr>
<td>Self study:</td>
<td></td>
<td>90h</td>
<td>60.00%</td>
</tr>
</tbody>
</table>
Content

1 - Differential equations

Description:
Introduction to ordinary differential equations (ODE): what they are, what they are good for, what a solution is. Examples in modelling.

- What is an ordinary differential equation (ODE).
- Solution of an ODE.
- Classification of ODE.
- Solution of ODE with separated variables.
- Problems in mechanics, electricity and population growth models.
- Numerical methods. Euler's method.

Related activities:
A1, A3, A4

Specific objectives:
The student will learn how to

- identify an ODE and classify it.
- check an ODE solution.
- compute the arbitrary constants in a solution, given appropriate initial conditions.
- recognize the ODE in simple problems from mechanics, electricity and population models.
- solve ODE with separated variables.
- explain the existence of different numerical methods for ODE, and the possible errors associated to their use.
- transform an arbitrary ODE to a form suitable for the numerical computation of the solutions.
- use MATLAB's commands and drivers for the numerical solution of ODE and the analysis of the solutions.
## 2 - Laplace transform

<table>
<thead>
<tr>
<th>Description:</th>
<th>Learning time: 40h</th>
</tr>
</thead>
<tbody>
<tr>
<td>How to use the main properties of the Laplace transform to solve linear ODE with constant coefficients.</td>
<td>Theory classes: 16h</td>
</tr>
<tr>
<td>· Laplace transform of an admissible function. First examples.</td>
<td>Self study : 24h</td>
</tr>
<tr>
<td>· Basic properties of the Laplace transform.</td>
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<tr>
<td>· Tables of Laplace transforms. Inverse Laplace transform.</td>
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<tr>
<td>· Further properties of the Laplace transform.</td>
<td></td>
</tr>
<tr>
<td>· Using the Laplace transform to solve linear ODE with constant coefficients.</td>
<td></td>
</tr>
<tr>
<td>· ODE with piecewise-defined right-hand sides. The Heaviside step function and the translation-in-time property.</td>
<td></td>
</tr>
<tr>
<td>· Systems of linear first-order ODE with constant coefficients. Resolution using the Laplace transform.</td>
<td></td>
</tr>
<tr>
<td>Characteristic polynomial of a system.</td>
<td></td>
</tr>
</tbody>
</table>

### Related activities:
A1, A4

### Specific objectives:
The student will be able to

- compute the Laplace transform of a function from the definition in simple cases.
- compute the Laplace transform of a function from known transforms and using the properties of the transform.
- compute inverse Laplace transforms using the properties of the transform and the partial fraction decomposition.
- compute the solution to linear ODE with constant coefficients using the Laplace transform.
- write a piecewise-defined function using the Heaviside function.
- compute the solution to linear constant-coefficient ODE with discontinuous with piecewise-defined right-hand sides.
- compute the solution to systems of first-order linear constant-coefficient ODE using the Laplace transform.
### 3 - Analysis of the solutions of linear systems

**Learning time:** 38h  
Theory classes: 12h  
Laboratory classes: 2h  
Self study: 24h

<table>
<thead>
<tr>
<th>Description</th>
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<tbody>
<tr>
<td>How to compute important properties, in the time and Laplace domains, of the solutions of linear constant-coefficient ODE (LCCODE), both for first-order and second-order equations and for first-order systems.</td>
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</tbody>
</table>

- General solution of a LCCODE.  
- Computation of a solution for a LCCODE with sinusoidal right-hand side using phasors.  
- Stability. Transient and stationary regimes. Characteristic time.  
- Mechanical and electrical examples.  
- Stability of LCCODE first-order systems.  
- Control systems. Transfer function and impulse response of a linear input/output system.

**Related activities:**  
A2, A3, A4

**Specific objectives:**  
The student will learn how to  
- compute the general solution of a LCCODE using the Laplace transform.  
- decide the stability of a first-order or second-order LCCODE, and compute and explain the meaning of the characteristic time for the stable case.  
- tell the stationary regime from the transient one for a stable LCCODE.  
- compute the stationary regime of a LCCODE with sinusoidal right-hand side using phasors.  
- decide the stability of first-order systems of LCCODE.  
- compute the transfer function and the impulse response of a linear control system.
4 - Fourier series

Description:
How to compute the Fourier series of a periodic function and how to understand it.

- Periodic signals.
- Fourier series in real and complex form.
- Spectrum of a periodic signal. Band-limited signals.
- Comments on the convergence of Fourier series. Gibbs' phenomenon.
- Discrete Fourier transform. Sampling and aliasing.

Related activities:
A2, A3, A4

Specific objectives:
The student will be able to

- compute the period of a signal, if any, and decide whether the signal is band-limited or not.
- compute the general term of the Fourier series of simple periodic functions, in real and complex form, and write down and simplify the series.
- identify the relative weight of the several harmonics in a periodic function and explain the effect of the jump discontinuities.
- compute, represent and understand the spectrum of a periodic function.
- recognize from the spectrum whether a signal is band-limited or not.
- use the FFT and recognize its problems.

Qualification system

A1=first partial exam (parts 1 and 2)
A2=second partial exam (parts 3 and 4)
A3= practical work with MATLAB/OCTAVE
A4=final exam (parts 1, 2, 3 and 4)
A4=laboratory exam with MATLAB/OCTAVE

FINAL GRADE = MAX (0.4*A1 + 0.4*A2 + 0.2*A3, 0.8*A4 + 0.2*A3)
(all scores on a 0-10 scale)

Regulations for carrying out activities

Activities A1, A2 and A4 are individual, written, problem-solving exams. Notes, books and digital devices are not permitted.
Activities A2 and A4 are simultaneous and incompatible.
Activity A3 is homework and can be done individually or in pairs.
Activity A4 is re-gradable.
Bibliography

Basic:


Complementary:


